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Projections of structure functions in a spin-one hadron

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There exist new polarized structure functions in a spin-one hadron. In deep inelastic electron scattering from a spin-one hadron, there are eight structure functions F_1, F_2, g_1, g_2, b_1, b_2, b_3, and b_4. We derive projections to these eight functions from the hadron tensor $W^{\mu\nu}$ by combinations of the hadron momentum and its polarization vectors.

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I. INTRODUCTION

High-energy spin physics became an important topic since the discovery of the European Muon Collaboration (EMC) spin effect [1]. It indicated that almost none of the nucleon spin is carried by quarks, whereas the total amount should be carried in a naive quark model. Since the EMC finding, many experiments have been done for the nucleon spin. We now have a rough idea about each quark-spin contribution to the nucleon spin [2]. However, contributions from gluon spin and orbital angular momenta are not still clear, so that various experimental studies are in progress for clarifying the origin of the nucleon spin.

From the studies of the spin-1/2 nucleon, we found that our native models cannot be applied to hadron spin physics. Therefore, it is important to test our knowledge on spin physics by other spin observables. There exist new polarized structure functions for spin-one hadrons [3, 4, 5]. There are also new fragmentation functions [6], generalized parton distributions [7], and studies of target mass corrections [8] for spin-one hadrons.

Among the new structure functions, the leading-twist ones are b_1 and b_2 which are related with each other by the Callan-Gross type relation in the Bjorken scaling limit [4]. It should be noted that b_1 vanishes if spin-1/2 constituents are in the orbital S state, so that it is sensitive to dynamical aspects of spin and orbital structure and possibly to non-nucleonic degrees of freedom, for example, in the deuteron. In conventional approaches, such tensor structure arises due to the D state admixture [3, 4], pions [9], and shadowing effects [10, 11] in a nucleus. However, as it became obvious in the nucleon spin, it is likely that high-energy tensor structure would not be simply described by such conventional models. The new structure functions could be important for probing unexplored dynamical aspects of hadron spin.

The first measurement of the structure function b_1 was made by the HERMES collaboration in 2005 [12]. The data indicated a finite distribution at $x < 0.1$, which roughly agrees with a double scattering contribution estimated in Ref. [11]. The data are also consistent with the quark-parton model sum rule for b_1 [13] although experimental errors are still large. Positivity constraints are studied for b_1 in Ref. [14]. In future, there are possibilities that the tensor structure functions could be investigated in various facilities such as Thomas Jefferson National Accelerator Facility (JLab) and Japan Proton Accelerator Research Complex (J-PARC) [15].

Although there are theoretical formalisms on the tensor structure functions in polarized electron-hadron scattering [3, 4, 5], polarized hadron-hadron reactions, such as at J-PARC, have not been well investigated for spin-one hadrons. New polarized structure functions were found in a general formalism of polarized proton-deuteron Drell-Yan processes by imposing time-reversal and parity invariances as well as Hermiticity [16]. It should be noted that the tensor distributions can be measured without polarizing the proton beam [16].

It is important to obtain a reliable theoretical predication for the tensor structure functions in order to compare with experimental measurements. In calculating structure functions for nuclei such as the deuteron, a convolution model is often used for the hadron tensor $W^{\mu\nu}$, which is given by the nucleonic tensor convoluted with the spectral function of a nucleon in a nucleus [17]. In order to extract each structure function from $W^{\mu\nu}$, a corresponding projection operator needs to be multiplied [18]. For the spin-$\frac{1}{2}$ nucleon, such projections have been already studied [18, 19]. It is the purpose of this article to derive projection operators for the structure functions of a spin-one hadron. Our results should be useful for future theoretical studies on spin-one hadrons.

This article consists of the following. In Sec. II, a general framework of the structure functions for a spin-one hadron is discussed. Then, the projection operators are obtained in Sec. III, and results are summarized in Sec. IV.

II. STRUCTURE FUNCTIONS OF SPIN-ONE HADRONS

A cross section for deep inelastic electron-hadron scattering is described in terms of a hadron tensor $W_{\mu\nu}$. The hadron tensor for a spin-$\frac{1}{2}$ target is expressed by four

structure functions F_1, F_2, g_1, and g_2 [20]:

$$W_{\mu\nu}^{\lambda_f\lambda_i} = -F_1\hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{p}_\mu\hat{p}_\nu + \frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q s^\sigma - s\cdot q p^\sigma), \quad (1)$$

where $\epsilon_{\mu\nu\lambda\sigma}$ is an antisymmetric tensor with the convention $\epsilon_{0123} = 1$, ν is defined by $\nu = p\cdot q/M$ with the hadron mass M, hadron momentum p, and momentum transfer q, Q^2 is given by $Q^2 = -q^2 > 0$, and s^μ is the spin vector [21] which satisfies $s\cdot p = 0$. In Eq. (1), we introduced notations:

$$\hat{g}_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \hat{a}_\mu \equiv a_\mu - \frac{a\cdot q}{q^2}q_\mu, \quad (2)$$

which ensure the current conservation $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$. The a^μ is a four vector, which is, for example, p^μ in Eq. (1). The coefficients of the unpolarized structure functions F_1 and F_2 are symmetric under $\mu\leftrightarrow\nu$ in $W_{\mu\nu}$, while those of the polarized structure functions g_1 and g_2 are antisymmetric.

In a spin-one hadron, there are four additional structure functions b_1, b_2, b_3 and b_4 in the hadron tensor [4, 20]:

$$\begin{aligned} W_{\mu\nu}^{\lambda_f\lambda_i} &= -F_1\hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{p}_\mu\hat{p}_\nu - b_1 r_{\mu\nu} \\ &+ \frac{1}{6}b_2(s_{\mu\nu}+t_{\mu\nu}+u_{\mu\nu}) + \frac{1}{2}b_3(s_{\mu\nu}-u_{\mu\nu}) + \frac{1}{2}b_4(s_{\mu\nu}-t_{\mu\nu}) \\ &+ \frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q s^\sigma - s\cdot q p^\sigma), \end{aligned} \quad (3)$$

where $r_{\mu\nu}$, $s_{\mu\nu}$, $t_{\mu\nu}$, and $u_{\mu\nu}$ are defined by

$$\begin{aligned} r_{\mu\nu} =& \frac{1}{\nu^2}\left[q\cdot E^*(\lambda_f)q\cdot E(\lambda_i) - \frac{1}{3}\nu^2\kappa\right]\hat{g}_{\mu\nu}, \\ s_{\mu\nu} =& \frac{2}{\nu^2}\left[q\cdot E^*(\lambda_f)q\cdot E(\lambda_i) - \frac{1}{3}\nu^2\kappa\right]\frac{\hat{p}_\mu\hat{p}_\nu}{M\nu}, \\ t_{\mu\nu} =& \frac{1}{2\nu^2}\Big[q\cdot E^*(\lambda_f)\left\{\hat{p}_\mu\hat{E}_\nu(\lambda_i) + \hat{p}_\nu\hat{E}_\mu(\lambda_i)\right\} \\ &+ \left\{\hat{p}_\mu\hat{E}^*_\nu(\lambda_f) + \hat{p}_\nu\hat{E}^*_\mu(\lambda_f)\right\}q\cdot E(\lambda_i) - \frac{4\nu}{3M}\hat{p}_\mu\hat{p}_\nu\Big], \\ u_{\mu\nu} =& \frac{M}{\nu}\Big[\hat{E}^*_\mu(\lambda_f)\hat{E}_\nu(\lambda_i) + \hat{E}^*_\nu(\lambda_f)\hat{E}_\mu(\lambda_i) \\ &+ \frac{2}{3}\hat{g}_{\mu\nu} - \frac{2}{3M^2}\hat{p}_\mu\hat{p}_\nu\Big], \end{aligned} \quad (4)$$

where $\kappa = 1 + Q^2/\nu^2$, and s^μ is the spin vector which satisfies $p\cdot s = 0$. The E^μ is the polarization vector of the spin-one hadron and it satisfies $p\cdot E = 0, E^*\cdot E = -1$. The initial and final spin states are denoted by λ_i and λ_f, respectively. Off-diagonal terms with $\lambda_f \neq \lambda_i$ need to be taken into account in the general case to include higher-twist contributions [4, 5]. The coefficients of b_1, b_2, b_3 and b_4 are symmetric under $\mu\leftrightarrow\nu$, and they vanish under the spin average. The functions F_1, F_2, g_1, and g_2 exist in a spin-$\frac{1}{2}$ hadron as shown in Eq. (1). The b_1, b_2, b_3 and b_4 are new structure functions for a spin-one hadron and they are associated with its tensor structure nature.

III. PROJECTIONS TO STRUCTURE FUNCTIONS

In the following calculations, we choose the frame in which the target is at rest and the photon is moving in the opposite direction to the z axis. Then, the target-hadron and virtual-photon momenta are given by $p^\mu = (M,0,0,0)$ and $q^\mu = (\nu,0,0,-|\vec{q}|)$, respectively. However, results are Lorentz invariant, so that they do not depend on the choice of the specific frame.

A. Spin-$\frac{1}{2}$ hadrons

Before discussing projections in a spin-one hadron, we first show the spin-$\frac{1}{2}$ case, in which the structure functions F_1, F_2, g_1, and g_2 exist. Projections to these functions from $W_{\mu\nu}$ are discussed. The hadron tensor $W_{\mu\nu}$ for a spin-$\frac{1}{2}$ hadron is given in Eq. (1). Such projections were discussed in other articles [18, 19]; however, they are explained in order to compare with the spin-one projections in Sec. III B.

The spin vector is given by $\vec{s}_{\lambda_f\lambda_i} = N_{\lambda_f\lambda_i}u^\dagger_{\lambda_f}\vec{\sigma}u_{\lambda_i}$ [4] where λ_1 and λ_2 are initial and final target spins, respectively, along the z axis, $\vec{\sigma}$ is the Pauli matrix, u_λ is the Pauli spinor, and $N_{\lambda_f\lambda_i}$ is a normalization factor to satisfy $(\vec{s}_{\lambda_f\lambda_i})^*\cdot\vec{s}_{\lambda_f\lambda_i} = 1$. The spin four-vector is then given by $s^\mu_{\lambda_f\lambda_i} = (0,\vec{s}_{\lambda_f\lambda_i})$ in the rest frame of the hadron. Using $u_\uparrow = \begin{pmatrix}1\\0\end{pmatrix}$ and $u_\downarrow = \begin{pmatrix}0\\1\end{pmatrix}$, we have explicit expressions for the spin vectors $s^\mu_{\uparrow\uparrow}$ and $s^\mu_{\uparrow\downarrow}$:

$$s^\mu_{\uparrow\uparrow} = (0,0,0,1), \quad s^\mu_{\uparrow\downarrow} = \frac{1}{\sqrt{2}}(0,1,-i,0), \quad s^\mu_{\downarrow\uparrow} = (s^\mu_{\uparrow\downarrow})^*. \quad (5)$$

In order to project out the four structure functions, four independent combinations of momentum and spin need to be used. We choose a set:

$$g^{\mu\nu}, \quad \frac{p^\mu p^\nu}{M^2}, \quad \frac{i}{M}\epsilon^{\mu\nu\alpha\beta}q_\alpha s^{\uparrow\uparrow}_\beta\delta_{\lambda_f\frac{1}{2}}\delta_{\lambda_i\frac{1}{2}}, \quad \frac{i}{M}\epsilon^{\mu\nu\alpha\beta}q_\alpha s^{\uparrow\downarrow}_\beta\delta_{\lambda_f-\frac{1}{2}}\delta_{\lambda_i\frac{1}{2}}, \quad (6)$$

for the projections from the hadron tensor $W_{\mu\nu}^{\lambda_f\lambda_i}$, which depends on the initial and final spins through the spin vector s^μ. Here, $\delta_{\lambda\lambda'} = 1$ (0) for $\lambda = \lambda'$ ($\lambda\neq\lambda'$). The first two terms in Eq. (6) are symmetric under the exchange $\mu\leftrightarrow\nu$ and they project out the symmetric parts (F_1 and F_2) of $W_{\mu\nu}^{\lambda_f\lambda_i}$. The latter two terms are antisymmetric and they project out the antisymmetric parts (g_1 and g_2). Any other combinations such as $p^\mu s^\nu_{\uparrow\uparrow} + p^\nu s^\mu_{\uparrow\uparrow}$ are not independent terms, so that there are only four independent terms as shown in Eq. (6). Using these terms, we can project out each structure function as follows:

$$F_1 = -\frac{1}{2}\left(g^{\mu\nu} - \frac{\kappa-1}{\kappa}\frac{p^\mu p^\nu}{M^2}\right)W^{\lambda_f\lambda_i}_{\mu\nu},$$

$$F_2 = -\frac{x}{\kappa}\left(g^{\mu\nu} - \frac{\kappa-1}{\kappa}\frac{3p^\mu p^\nu}{M^2}\right)W^{\lambda_f\lambda_i}_{\mu\nu},$$

$$g_1 = -\frac{i}{2\kappa\nu}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(s^{\uparrow\uparrow}_\beta\delta_{\lambda_f\frac{1}{2}}\delta_{\lambda_i\frac{1}{2}} - s^{\uparrow\downarrow}_\beta\delta_{\lambda_f-\frac{1}{2}}\delta_{\lambda_i\frac{1}{2}}\right)W^{\lambda_f\lambda_i}_{\mu\nu},$$

$$g_2 = \frac{i}{2\kappa\nu}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(s^{\uparrow\uparrow}_\beta\delta_{\lambda_f\frac{1}{2}}\delta_{\lambda_i\frac{1}{2}} + \frac{s^{\uparrow\downarrow}_\beta}{\kappa-1}\delta_{\lambda_f-\frac{1}{2}}\delta_{\lambda_i\frac{1}{2}}\right)W^{\lambda_f\lambda_i}_{\mu\nu}, \tag{7}$$

where $x = Q^2/(2p\cdot q)$, and summations are taken over λ_i and λ_f in g_1 and g_2.

B. Spin-1 hadrons

Next, we derive projections to the structure functions for a spin-1 target. We use the following spherical unit vectors as the target polarization vector [22]:

$$E^\mu(\lambda=\pm1) = \frac{1}{\sqrt{2}}(0,\mp1,-i,0),$$
$$E^\mu(\lambda=0) = (0,0,0,1). \tag{8}$$

For a spin-one hadron, the spin vector is expressed by the polarization vector E^μ as [4]

$$(s^{\lambda_f\lambda_i})^\mu = -\frac{i}{M}\epsilon^{\mu\nu\alpha\beta}E^*_\nu(\lambda_f)E_\alpha(\lambda_i)p_\beta, \tag{9}$$

where M is the mass of the spin-1 hadron. The initial and final polarization vectors are denoted by $E^\mu(\lambda_i)$ and $E^\mu(\lambda_f)$, respectively, with the spin states λ_i and λ_f. The spin vector s^μ is then given for s^μ_{11}, s^μ_{10}, and s^μ_{01}:

$$s^\mu_{11} = (0,0,0,1),\quad s^\mu_{10} = \frac{1}{\sqrt{2}}(0,1,-i,0),\quad s^\mu_{01} = (s^\mu_{10})^*. \tag{10}$$

In projecting out the spin-1 structure functions, we may use only $\lambda=1$ and $\lambda=0$ terms because $\lambda=-1$ terms make the same contributions as $\lambda=1$ ones. We follow a similar procedure for the projections as the spin-$\frac{1}{2}$ case. In the spin-one case, we have eight independent structure functions, so that eight combinations need to be taken. We choose the following terms

$$g^{\mu\nu}\delta_{\lambda_f1}\delta_{\lambda_i1},\quad g^{\mu\nu}\delta_{\lambda_f0}\delta_{\lambda_i0},\quad g^{\mu\nu}\delta_{\lambda_f1}\delta_{\lambda_i0},$$
$$\frac{p^\mu p^\nu}{M^2}\delta_{\lambda_f1}\delta_{\lambda_i1},\quad \frac{p^\mu p^\nu}{M^2}\delta_{\lambda_f0}\delta_{\lambda_i0},$$
$$\frac{1}{M}\left[p^\mu E^\nu(\lambda=1) + p^\nu E^\mu(\lambda=1)\right]\delta_{\lambda_f1}\delta_{\lambda_i0},$$
$$\frac{i}{M}\epsilon^{\mu\nu\alpha\beta}q_\alpha s^{11}_\beta\delta_{\lambda_f1}\delta_{\lambda_i1},\quad \frac{i}{M}\epsilon^{\mu\nu\alpha\beta}q_\alpha s^{10}_\beta\delta_{\lambda_f0}\delta_{\lambda_i1}. \tag{11}$$

The first six terms are associated with projections to the structure functions F_1, F_2, b_1, b_2, b_3 and b_4, and the last two terms are to g_1 and g_2. Off-diagonal terms with $\lambda_f \neq \lambda_i$ are needed for including higher-twist effects. Using these eight independent terms, we obtain the projections from the hadron tensor in Eq. (3) as

$$F_1 = -\frac{1}{2}\left(g^{\mu\nu} - \frac{\kappa-1}{\kappa}\frac{p^\mu p^\nu}{M^2}\right)\frac{1}{3}\delta_{\lambda_f\lambda_i}W^{\lambda_f\lambda_i}_{\mu\nu},\qquad F_2 = -\frac{x}{\kappa}\left(g^{\mu\nu} - \frac{\kappa-1}{\kappa}\frac{3p^\mu p^\nu}{M^2}\right)\frac{1}{3}\delta_{\lambda_f\lambda_i}W^{\lambda_f\lambda_i}_{\mu\nu},$$

$$g_1 = -\frac{i}{2\kappa\nu}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(s^{11}_\beta\delta_{\lambda_f1}\delta_{\lambda_i1} - s^{10}_\beta\delta_{\lambda_f0}\delta_{\lambda_i1}\right)W^{\lambda_f\lambda_i}_{\mu\nu},\qquad g_2 = \frac{i}{2\kappa\nu}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(s^{11}_\beta\delta_{\lambda_f1}\delta_{\lambda_i1} + \frac{s^{10}_\beta}{\kappa-1}\delta_{\lambda_f0}\delta_{\lambda_i1}\right)W^{\lambda_f\lambda_i}_{\mu\nu},$$

$$b_1 = \left[-\frac{1}{2\kappa}g^{\mu\nu}\left(\delta_{\lambda_f0}\delta_{\lambda_i0} - \delta_{\lambda_f1}\delta_{\lambda_i0}\right) + \frac{\kappa-1}{2\kappa^2}\frac{p^\mu p^\nu}{M^2}\left(\delta_{\lambda_f0}\delta_{\lambda_i0} - \delta_{\lambda_f1}\delta_{\lambda_i1}\right)\right]W^{\lambda_f\lambda_i}_{\mu\nu},$$

$$\begin{aligned}b_2 = \frac{x}{\kappa^2}\Bigg[&g^{\mu\nu}\Big\{-\delta_{\lambda_f0}\delta_{\lambda_i0} - 2(\kappa-1)\delta_{\lambda_f1}\delta_{\lambda_i1} + (2\kappa-1)\delta_{\lambda_f1}\delta_{\lambda_i0}\Big\} + \frac{3(\kappa-1)}{\kappa}\frac{p^\mu p^\nu}{M^2}\left(\delta_{\lambda_f0}\delta_{\lambda_i0} - \delta_{\lambda_f1}\delta_{\lambda_i1}\right)\\ &- \frac{4(\kappa-1)}{\sqrt{\kappa}M}\left\{p^\mu E^\nu(\lambda=1) + p^\nu E^\mu(\lambda=1)\right\}\delta_{\lambda_f1}\delta_{\lambda_i0}\Bigg]W^{\lambda_f\lambda_i}_{\mu\nu},\end{aligned}$$

$$\begin{aligned}b_3 = \frac{x}{3\kappa^2}\Bigg[&g^{\mu\nu}\Big\{-\delta_{\lambda_f0}\delta_{\lambda_i0} + \frac{2(2\kappa^2+2\kappa-1)}{\kappa-1}\delta_{\lambda_f1}\delta_{\lambda_i1} - \frac{4\kappa^2+3\kappa-1}{\kappa-1}\delta_{\lambda_f1}\delta_{\lambda_i0}\Big\} + \frac{3(\kappa-1)}{\kappa}\frac{p^\mu p^\nu}{M^2}\left(\delta_{\lambda_f0}\delta_{\lambda_i0} - \delta_{\lambda_f1}\delta_{\lambda_i1}\right)\\ &- \frac{4(\kappa-1)}{\sqrt{\kappa}M}\left\{p^\mu E^\nu(\lambda=1) + p^\nu E^\mu(\lambda=1)\right\}\delta_{\lambda_f1}\delta_{\lambda_i0}\Bigg]W^{\lambda_f\lambda_i}_{\mu\nu},\end{aligned}$$

$$\begin{aligned}b_4 = \frac{x}{3\kappa^2}\Bigg[&g^{\mu\nu}\Big\{-\delta_{\lambda_f0}\delta_{\lambda_i0} - \frac{2(\kappa^2+4\kappa+1)}{\kappa-1}\delta_{\lambda_f1}\delta_{\lambda_i1} + \frac{2\kappa^2+9\kappa+1}{\kappa-1}\delta_{\lambda_f1}\delta_{\lambda_i0}\Big\} + \frac{3(\kappa-1)}{\kappa}\frac{p^\mu p^\nu}{M^2}\left(\delta_{\lambda_f0}\delta_{\lambda_i0} - \delta_{\lambda_f1}\delta_{\lambda_i1}\right)\\ &+ \frac{4(2\kappa+1)}{\sqrt{\kappa}M}\left\{p^\mu E^\nu(\lambda=1) + p^\nu E^\mu(\lambda=1)\right\}\delta_{\lambda_f1}\delta_{\lambda_i0}\Bigg]W^{\lambda_f\lambda_i}_{\mu\nu},\end{aligned} \tag{12}$$

where summations are taken over λ_i and λ_f.

It is useful to show the projections also in the Bjorken scaling limit, $\nu, Q^2 \to \infty$ at finite $x = Q^2/(2p\cdot q)$, because the leading-twist structure functions such as b_1 and b_2 are first investigated experimentally. In the scaling limit, we have the following relations

$$\lim_{\mathrm{Bj}} g^{\mu\nu}W^{10}_{\mu\nu} = \lim_{\mathrm{Bj}} g^{\mu\nu}W^{11}_{\mu\nu},\qquad \lim_{\mathrm{Bj}}(\kappa-1)\frac{p^\mu p^\nu}{M^2}W^{\lambda\lambda}_{\mu\nu} = 0,$$
$$\lim_{\mathrm{Bj}}\frac{\kappa-1}{M}\left[p^\mu E^\nu(\lambda=1) + p^\nu E^\mu(\lambda=1)\right]W^{10}_{\mu\nu} = 0, \tag{13}$$

by noting $\kappa \to 1$, $2xF_1 \to F_2$, and. $2xb_1 \to b_2$. Then, we obtain the following expressions for the leading-twist structure functions:

$$
\begin{aligned}
F_1 =& \frac{1}{2x}F_2 = -\frac{1}{2}g^{\mu\nu}\frac{1}{3}\delta_{\lambda_f\lambda_i}W_{\mu\nu}^{\lambda_f\lambda_i}, \\
g_1 =& -\frac{i}{2\nu}\epsilon^{\mu\nu\alpha\beta}q_\alpha s_\beta^{11}\delta_{\lambda_f 1}\delta_{\lambda_i 1}W_{\mu\nu}^{\lambda_f\lambda_i}, \\
b_1 =& \frac{1}{2x}b_2 = \frac{1}{2}g^{\mu\nu}\left(\delta_{\lambda_f 1}\delta_{\lambda_i 1} - \delta_{\lambda_f 0}\delta_{\lambda_i 0}\right)W_{\mu\nu}^{\lambda_f\lambda_i}, \qquad (14)
\end{aligned}
$$

in the Bjorken scaling limit.

We have derived projections into the structure functions of a spin-one hadron from its hadron tensor $W_{\mu\nu}$. If a model is built for the hadron tensor, all the structure functions are derived without an approximation by using the projections in Eq. (12).

The spin structure of a spin-one hadron should be investigated in future measurements. It should be possible to measure b_1 in the large-x region at electron facilities such as JLab. The polarized proton-deuteron Drell-Yan processes should be valuable in probing antiquark tensor polarizations as unpolarized Drell-Yan measurements played a key role in finding flavor dependence of antiquark distributions in the nucleon [13, 23]. The measurement of the tensor polarized antiquark distributions is possible without proton polarization [16]. There is a possibility to investigate such Drell-Yan processes at J-PARC [15]. As the unexpected EMC spin measurement led to many investigations on spin structure of spin-$\frac{1}{2}$ hadron, the tensor polarization studies could lead to a new dynamical aspect of hadron spin physics.

IV. SUMMARY

Spin structure of a spin-one hadron is interesting as a future research topic because there exist new tensor structure functions which do not appear in the spin-$\frac{1}{2}$ nucleon. There are eight structure functions, F_1, F_2, g_1, g_2, b_1, b_2, b_3, and b_4, in electron scattering from a spin-one hadron. In this article, the projection operators have been derived for the structure functions of a spin-one hadron by using combination of its momentum, polarization, and spin vectors. They are useful in theoretical calculations because the structure functions need to be extracted from a calculated hadron tensor $W_{\mu\nu}$ in theoretical models.

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